Hong Kong Mathematics Olympiad (1993 – 94)

Final Event – Sample (Group)

香港数学竞赛 (1993 – 94)

决赛项目– 样本 (团体)

(i) If $x * y = xy + 1$ and $a = (2 * 4) * 2$, find a.

若 $x * y = xy + 1$，且 $a = (2 * 4) * 2$，求 a。

$a =$

(ii) If the b^{th} prime number is a, find b.

若第 b 个质数为 a，求 b。

$b =$

(iii) If $c = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\cdots\left(1 - \frac{1}{50}\right)$, find c in the simplest fractional form.

若 $c = \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\cdots\left(1 - \frac{1}{50}\right)$，试以最简单之分数表 c。

$c =$

(iv) If d is the area of a square inscribed in a circle of radius 10, find d.

一正方形内接于一个半径为 10 之圆。若正方形之面积为 d，求 d。

$d =$

Hong Kong Mathematics Olympiad (1993 – 94)
Final Event 6 (Group)
香港数学竞赛 (1993 – 94)
决赛项目 6 (团体)

(i) If $\log_2 a - 2\log_a 2 = 1$, find a.

若 $\log_2 a - 2\log_a 2 = 1$，求 a。

$a =$

(ii) If $b = \log_3\left[2(3+1)\left(3^2+1\right)\left(3^4+1\right)\left(3^8+1\right)+1\right]$, find b.

若 $b = \log_3\left[2(3+1)\left(3^2+1\right)\left(3^4+1\right)\left(3^8+1\right)+1\right]$，求 b。

$b =$

(iii) If a 31-day month is taken at random, find c, the probability that there are 5 Sundays in the month.

若任意选择一个有三十一日的月份，求该月有五个星期天的机率 c。

$c =$

(iv) A group of 5 people is to be selected from 6 men and 4 women. Find d, the number of ways that there are always more men than women.

从六名男士及四名女士中选出五人，组成一组。若其间共有 d 种选法，使男士必多于女士，求 d。

$d =$

Hong Kong Mathematics Olympiad (1993 – 94)

Final Event 7 (Group)

香港数学竞赛 (1993 – 94)

决赛项目 7 (团体)

(i) There are a zeros at the end of the product $1 \times 2 \times 3 \times \cdots \times 100$. Find a.

在$1 \times 2 \times 3 \times \cdots \times 100$ 的积数中，最末的 a 个位都是 0。求 a。

$a =$

(ii) Find b, if b is the remainder when 1998^{10} is divided by 10^4.

1998^{10} 除以 10^4，所得余数为 b，求 b。

$b =$

(iii) Find the largest value of c, if $c = 2 - x + 2\sqrt{x-1}$ and $x > 1$.

若 $c = 2 - x + 2\sqrt{x-1}$ 且 $x > 1$，求 c 之最大值。

$c =$

(iv) Find the least value of d, if $\left|\frac{3-2d}{5} + 2\right| \le 3$.

若 $\left|\frac{3-2d}{5} + 2\right| \le 3$，求 d 的最小值。

$d =$

Hong Kong Mathematics Olympiad (1993 – 94)
Final Event 8 (Group)
香港数学竞赛 (1993 – 94)
决赛项目 8 (团体)

(i) From 1 to 121, there are a numbers which are multiplies of 3 or 5. Find a.

由 1 至 121，有 a 个数是 3 或是 5 的倍数。求 a。

$a =$

(ii) From 1 to 121, there are b numbers which are not divisible by 5 or 7. Find b.

由 1 至 121，有 b 个数不能被 5 或 7 整除。求 b。

$b =$

From the digits 1, 2, 3, 4, when each digit can be used repeatedly, 4-digit numbers are formed. Find

用 1、2、3、4 这四个数字，而每个数字均可重复使用，则可组成一些 4 位数。求

(iii) c, the number of 4-digit numbers that can be formed.

共可组成的 4 位数的个数 c。

$c =$

(iv) d, the sum of all these 4-digit numbers.

所组成的 4 位数的总和 d。

$d =$

Hong Kong Mathematics Olympiad (1993 – 94)

Final Event 9 (Group)

香港数学竞赛 (1993 – 94)

决赛项目 9 (团体)

A , B , C , D are different integers ranging from 0 to 9 and

A 、 B 、 C 、 D 为由 0 至 9 间的不同整数，而

$$\begin{array}{r} A\ B\ A \\ \times \qquad A\ B\ A \\ \hline C\ C\ D\ C\ C \end{array}$$

Find A , B , C , D .

求 A、B、C、D。

A =	B =	C =	D =

Hong Kong Mathematics Olympiad (1993 – 94)
Final Event 10 (Group)
香港数学竞赛 (1993 – 94)
决赛项目 10 (团体)

In rectangle $ABCD$, $AD = 10$, $CD = 15$, P is a point inside the rectangle such that $PB = 9$, $PA = 12$. Find

在长方形 $ABCD$ 中，$AD = 10$，$CD = 15$，P 为长方形内一点，使 $PB = 9$，$PA = 12$。求

(i) a, the length of PD and

PD 之长 a，及

$a =$

(ii) b, the length of PC.

PC 之长 b。

$b =$

(iii) It is given that $\sin 2\theta = 2\sin\theta\cos\theta$. Find c, if

已知 $\sin 2\theta = 2\sin\theta\cos\theta$。求 c，若

$$c = \frac{\sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ}{\sin 160^\circ}$$

$c =$

(iv) It is given that $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$. Find d, if

已知 $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$，求 d，若

$$d = (1+\tan 21^\circ)(1+\tan 22^\circ)(1+\tan 23^\circ)(1+\tan 24^\circ)$$

$d =$